Time-Dependent Ginzburg-Landau Equations for Mixed d- and s-Wave Superconductors

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Abstract

A set of coupled time-dependent Ginzburg-Landau equations (TDGL) for superconductors of mixed d- and s-wave symmetry are derived microscopically from the Gor'kov equations by using the analytical continuation technique. The scattering effects due to impurities with both nonmagnetic and magnetic interactions are considered. We find that the d- and s-wave components of the order parameter can have very different relaxation times in the presence of nonmagnetic impurities. This result is contrary to a set of phenomenologically proposed TDGL equations and thus may lead to new physics in the dynamics of flux motion.

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I. INTRODUCTION

There are growing experimental evidences to suggest that high-T_c superconductors have a dominant $d_{x^2-y^2}$-wave pairing symmetry.[1] Based on symmetry considerations, Volovik[2] argued that an s-wave component of the order parameter should be generated near the core region of a vortex in a d-wave superconductor. This conclusion was later confirmed by a numerical calculation[3] and by studying a set of microscopically-derived two-component Ginzburg-Landau (GL) equations.[4]

In view of the enormous success of the GL theory for describing the equilibrium properties of superconductors near T_c, it is natural to generalize it to time-dependent situations. This generalization has become particularly desirable, since a set of phenomenological time-dependent Ginzburg-Landau (TDGL) equations for coupled s- and d-wave superconducting order parameters has been recently proposed,[5] and used to investigate the dynamics of vortices in high-T_c superconductors. One would very much like to know how valid is such an approach.

It is well known, however, that TDGL equations are not as universal in form as its time-independent variety, but can be dependent strongly on whether the system is gapful or gapless, and in the later case, whether a strong or weak gaplessness condition is assumed. The simplest set of TDGL equations for conventional s-wave superconductors was proposed phenomenologically by Schmid,[6] and subsequently derived microscopically by Gor'kov and Éliashberg[7] under the assumption of a strong gaplessness condition (*i.e.*, $\tau_s T_c \ll 1$, where τ_s is the spin-flip lifetime and T_c is the transition temperature). This set of equations has been used in the past to study the vortex dynamics in conventional superconductors.[8] Éliashberg[9] has later derived a more complex set of TDGL equations for low-T_c s-wave superconductors assuming only the weak gaplessness condition [*i.e.*, $\tau_s \Delta_0 \ll 1$, where $\Delta_0(T)$ is the equilibrium value of the (s-wave) order parameter in the absence of fields]. It has been used to study flux-flow resistivity[10] and the transport entropy of vortices.[11] Even more complex sets of TDGL equations have been derived subsequently assuming only the dirty limit condition ($\tau_1 T_c \ll 1$, where τ_1 is the total scattering lifetime) and $(1 - T/T_c) \ll 1$,[12] so the system need no longer be gapless. However, this set of equations is so complex that it has not yet been extensively used.

In this work, we shall derive microscopically a set of coupled TDGL equations for superconductors with mixed d- and s-wave pairing symmetry based on the approach of Gor'kov and Éliashberg[7,9] in the presence of impurities with both spin-flip and non-spin-flip interactions and assuming only weak gaplessness conditions for both waves (*i.e.*, $\tau_1 \Delta_{d0} \ll 1$, and $\tau_s \Delta_{s0} \ll 1$.) The primary objective of this derivation is to establish a reasonably reliable set of equations governing the dynamics of coupled d- and s-wave order parameters which are hopefully valid for describing the dynamic properties of high-T_c superconductors.

The outline of this paper is as follows: In Sec. II, the TDGL equations for the order parameters are derived. The expressions for current and charge density are presented in Sec. III. Finally, discussions and summary are given in Sec. IV.

II. TIME-DEPENDENT GINZBURG-LANDAU EQUATIONS FOR THE ORDER PARAMETERS

We begin with the Gor'kov equations:[13]

$$[-\tfrac{\partial}{\partial\tau} - h(\mathbf{x}\tau)]G_{\alpha\beta}(\mathbf{x}\tau,\mathbf{x}'\tau') - U_{\alpha\gamma}(\mathbf{x})G_{\gamma\beta}(\mathbf{x}\tau,\mathbf{x}'\tau')$$

$$+ \int d\mathbf{x}''\Delta_{\alpha\gamma}(\mathbf{x}\tau^{0+},\mathbf{x}''\tau)F^{\dagger}_{\gamma\beta}(\mathbf{x}''\tau,\mathbf{x}'\tau') = \delta(\mathbf{x}-\mathbf{x}')\delta(\tau-\tau')\delta_{\alpha\beta}\ , \tag{2.1a}$$

$$[\tfrac{\partial}{\partial\tau} - h^*(\mathbf{x}\tau)]F^{\dagger}_{\alpha\beta}(\mathbf{x}\tau,\mathbf{x}'\tau') - U_{\gamma\alpha}(\mathbf{x})F^{\dagger}_{\gamma\beta}(\mathbf{x}\tau,\mathbf{x}'\tau')$$

$$- \int d\mathbf{x}''\Delta^{\dagger}_{\alpha\gamma}(\mathbf{x}\tau^{0+},\mathbf{x}''\tau)G_{\gamma\beta}(\mathbf{x}''\tau,\mathbf{x}'\tau') = 0\ . \tag{2.1b}$$

Here repeated spin indices mean summing over these indices. In these equations,

$$h(\mathbf{x}\tau) = \frac{[\mathbf{p}+e\mathbf{A}(\mathbf{x}\tau)]^2}{2m} - e\varphi(\mathbf{x}\tau) - \mu\ , \tag{2.2}$$

is the single-electron ($-e$) Hamiltonian with $\mathbf{A}(\mathbf{x}\tau)$, $\varphi(\mathbf{x}\tau)$, and μ denoting the vector, scalar, and chemical potentials. (We have assumed $\hbar = c = 1$.) By assuming zero-range interactions bewteen electrons and impurities, the impurity scattering potential can be written as

$$U_{\alpha\beta}(\mathbf{x}) = \sum_{i\in I}[U_1\delta_{\alpha\beta} + U_2(\mathbf{S}_i\cdot\frac{\boldsymbol{\sigma}_{\alpha\beta}}{2})]\delta(\mathbf{x}-\mathbf{R}_i)\ , \tag{2.3}$$

where I denotes the set of impurity sites, $\boldsymbol{\sigma}$ is made of the Pauli spin matrices, $\mathbf{S}_i$ is the spin carried by an impurity at $\mathbf{R}_i$. U_1 and U_2 are the non-spin-flip and spin-flip interaction strengths, respectively. By definition, the order parameter in real coordinate and imaginary time space is

$$\Delta^*_{\alpha\beta}(\mathbf{x}\tau,\mathbf{x}'\tau) = V(\mathbf{x}-\mathbf{x}')F^{\dagger}_{\alpha\beta}(\mathbf{x}\tau^{0+},\mathbf{x}'\tau)\ , \tag{2.4}$$

where $-V(\mathbf{x}-\mathbf{x}')$ is the effective pairing interaction between electrons. Because of the spatial and temporal non-uniformity, the Green function $G_{\alpha\beta}(\mathbf{x}\tau,\mathbf{x}'\tau')$ and $F^{\dagger}_{\alpha\beta}(\mathbf{x}\tau,\mathbf{x}'\tau')$ are not the functions of coordinate and time differences. When expressed in the imaginary frequency space after the Fourier transform, they depend on two frequency variables. For the spatial coordinate dependence, as treated in the static case,[4] we express these two functions in terms of the center-of-mass coordinate $\mathbf{R} = (\mathbf{x}+\mathbf{x}')/2$ and the relative momentum after a Fourier transform with respect to the relative coordinate $\mathbf{r} = \mathbf{x}-\mathbf{x}'$. Thus Eq. (2.4) can be rewritten as:

$$\Delta^*_{\alpha\beta}(\mathbf{R},\mathbf{k};\omega) = T\sum_{\epsilon}\int\frac{d\mathbf{k}'}{(2\pi)^2}V(\mathbf{k}-\mathbf{k}')F^{\dagger}_{\alpha\beta}(\mathbf{R},\mathbf{k}';\epsilon,\epsilon-\omega)\ , \tag{2.5}$$

where $\mathbf{k}$ is the relative momentum, $\omega = 2in\pi T$ and $\epsilon = i(2n'+1)\pi T$ with integers n and n', $-V(\mathbf{k}-\mathbf{k}')$ is the pairing interaction in the momentum space, $F^{\dagger}(\mathbf{R},\mathbf{k};\epsilon,\epsilon')$ is the Fourier transform of $F^{\dagger}(\mathbf{x}\tau,\mathbf{x}'\tau')$. To relate to high-$T_c$ superconductors, we have assumed that the system under consideration is two dimensional. For the spin-singlet pairing, the order parameter is given in the spin space as $\Delta^*_{\alpha\beta} = \Delta^* g_{\alpha\beta}$, where

$$g_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}_{\alpha\beta}\ . \tag{2.6}$$

To obtain the TDGL equations for superconductors of a mixed d- and s-wave symmetry, we make the following ansatz for the pairing interaction and the order parameter

$$V(\mathbf{k}-\mathbf{k}') = V_s + V_d(\hat{k}_x^2 - \hat{k}_y^2)(\hat{k}_x'^2 - \hat{k}_y'^2) \ , \tag{2.7}$$

$$\Delta^*(\mathbf{R},\mathbf{k};\omega) = \Delta_s^*(\mathbf{R};\omega) + \Delta_d^*(\mathbf{R};\omega)(\hat{k}_x^2 - \hat{k}_y^2) \ , \tag{2.8}$$

where V_d and V_s are positive so that both the d- and s-channel interactions are attractive. The d-channel attractive interaction could originate from the antiferromagnetic spin fluctuation, whereas the s-channel attractive interaction might arise from phonon mediation.

Introducing the Green function $\tilde{G}^0$ of the normal metal, which satisfies the equation

$$[-\frac{\partial}{\partial\tau} - h(\mathbf{x}\tau)]\tilde{G}^0_{\alpha\beta}(\mathbf{x}\tau,\mathbf{x}'\tau') - U_{\alpha\gamma}(\mathbf{x})\tilde{G}^0_{\gamma\beta}(\mathbf{x}\tau,\mathbf{x}'\tau') = \delta(\mathbf{x}-\mathbf{x}')\delta(\tau-\tau')\delta_{\alpha\beta} \ , \tag{2.9}$$

Eq. (2.1) may be converted to a set of coupled integral equations:

$$\begin{aligned} G_{\alpha\beta}(\mathbf{x}\tau,\mathbf{x}'\tau') &= \tilde{G}^0_{\alpha\beta}(\mathbf{x}\tau,\mathbf{x}'\tau') \\ &\quad - \int d\mathbf{x}_1 d\mathbf{x}_2 d\tau_1 \tilde{G}^0_{\alpha\mu}(\mathbf{x}\tau,\mathbf{x}_1\tau_1)\Delta_{\mu\nu}(\mathbf{x}_1\tau_1,\mathbf{x}_2\tau_1)F^\dagger_{\nu\beta}(\mathbf{x}_2\tau_1,\mathbf{x}'\tau') \ , \end{aligned} \tag{2.10}$$

$$F^\dagger_{\alpha\beta}(\mathbf{x}\tau,\mathbf{x}'\tau') = \int d\mathbf{x}_1 d\mathbf{x}_2 d\tau_1 \tilde{G}^0_{\mu\alpha}(\mathbf{x}_1\tau_1,\mathbf{x}\tau)\Delta^*_{\mu\nu}(\mathbf{x}_1\tau_1,\mathbf{x}_2\tau_1)G_{\nu\beta}(\mathbf{x}_2\tau_1,\mathbf{x}'\tau') \ . \tag{2.11}$$

Also note that the normal-state Green function can in turn be written as an integral equation

$$\begin{aligned} \tilde{G}^0_{\alpha\beta}(\mathbf{x}\tau,\mathbf{x}'\tau') &= G^0_{\alpha\beta}(\mathbf{x}\tau,\mathbf{x}'\tau') + \int d\mathbf{x}'' d\tau'' G^0_{\alpha\gamma}(\mathbf{x}\tau,\mathbf{x}''\tau'') \\ &\quad \times [\frac{e\mathbf{A}(\mathbf{x}''\tau'')\cdot\mathbf{p}_{\mathbf{x}''}}{m} - e\varphi(\mathbf{x}''\tau'')]\tilde{G}^0_{\gamma\beta}(\mathbf{x}''\tau'',\mathbf{x}'\tau') \ , \end{aligned} \tag{2.12}$$

with G^0 as the normal-state single-particle Green function in the absence of the electromagnetic field but including the effect due to impurity scatterings. To write down the above integral equation, the squared term of the vector potential has been neglected and the Coulomb gauge is chosen.

A. Analytical continuation

To incorporate the time dependence of physical quantities, we use the analytical continuation technique discussed in Refs.[7,9] to transform imaginary frequencies into real frequencies. The procedure is as follows: (i) In Eq. (2.5), each term of the summation over the imaginary frequency ϵ can be regarded as the residue of an integral along the contour around the point $z=\epsilon$ so that we have the transformation $T\sum_\epsilon \to \frac{1}{4\pi i}\oint_C dz \tanh\frac{z}{2T}$. Associated with this transformation, all involved ϵ are replaced with z. For example,

$$T\sum_\epsilon G^0(-\epsilon)G^0(\epsilon-\omega') = \frac{1}{4\pi i}\oint_C dz \tanh\frac{z}{2T} G^0(-z)G^0(z-\omega') \ , \tag{2.13}$$

where the spatial and spin variables have been suppressed for simplicity. (ii) Deform the contour integral around z into the straight line integrals along $z = \epsilon \pm i0^+$, $z = \epsilon + \omega' \pm i0^+, \dots$, where $\epsilon \in (-\infty, \infty)$ is the real integral variable and 0^+ is infinitesimal. So far, $\omega', \dots$ are still imaginary frequency $2n\pi iT$ and we take $n \geq 0$ since we will perform the analytical continuation from the upper half-plane. As a consequence, each Green function $G^0(z)$ with energy variable coincident with this line is decomposed into $G^{0(R)}(\epsilon) - G^{0(A)}(\epsilon)$, where $G^{0(R,A)}$ are the retarded and advanced Green functions, respectively. The minus sign before $G^{0(A)}$ comes from changing the direction of integration. The other Green functions are mapped to the retarded or advanced Green function depending on their energy variable. Then Eq. (2.13) becomes

$$\begin{aligned}&\tfrac{1}{4\pi i}\textstyle\int_{-\infty}^{\infty} d\epsilon \tanh \tfrac{\epsilon}{2T}\{[G^{0(R)}(-\epsilon) - G^{0(A)}(-\epsilon)]G^{0(A)}(\epsilon - \omega') \\ &\quad + G^{0(R)}(-(\epsilon + \omega'))[G^{0(R)}(\epsilon) - G^{0(A)}(\epsilon)]\} \ .\end{aligned} \tag{2.14}$$

(iii) Since all ω', $\dots$, lie in the upper half-plane, the Fourier transform of the expansions (2.10) and (2.11) are analytical in each of these variables. Therefore, we can implement the analytical continuation by simply replacing all ω^i with $\omega^i + i0^+$. Simultaneously the discrete summation $T\sum_{\omega'}$ is replaced by a continuous integral $(2\pi)^{-1}\int d\omega'$ and the Kronecker delta $T^{-1}\delta_{\omega,\omega'}$ by the Dirac delta $(2\pi)\delta(\omega - \omega')$. Finally, we get

$$\begin{aligned}&\tfrac{1}{4\pi i}\textstyle\int_{-\infty}^{\infty} d\epsilon[-\tanh\tfrac{\epsilon}{2T} G^{0(A)}(-\epsilon)G^{0(A)}(\epsilon - \omega') + \tanh\tfrac{\epsilon-\omega'}{2T} G^{0(R)}(-\epsilon)G^{0(R)}(\epsilon - \omega')] \\ &\quad + \tfrac{1}{4\pi i}\textstyle\int_{-\infty}^{\infty} d\epsilon G^{0(R)}(-\epsilon)[\tanh\tfrac{\epsilon}{2T} - \tanh\tfrac{\epsilon-\omega'}{2T}]G^{0(A)}(\epsilon - \omega') \ .\end{aligned} \tag{2.15}$$

The first two terms consist of the only advanced and the only retarded Green functions (Following Ref.[7], we shall refer to them as the normal part). The rest part has those terms involving the product of retarded and advanced Green functions, in which a change from a retarded to an advanced Green function occurs in only one place (We shall refer to them as the anomalous parts). After obtaining the results in terms of real frequency, we can perform the inverse Fourier transform to represent them in a real time.

Using these rules, we can obtain the following expression

$$\begin{aligned}T\sum_{\epsilon} F^{\dagger}(\epsilon, \epsilon - \omega) \to\ & \frac{1}{4\pi i}\int d\epsilon \tanh\frac{\epsilon}{2T}[F^{\dagger(R)}(\epsilon + \omega, \epsilon) - F^{\dagger(A)}(\epsilon, \epsilon - \omega)] \\ & -\frac{1}{4\pi i}\int d\epsilon \int d\epsilon_1\{[G^{+(R)}(\epsilon, \epsilon_1)\Delta^*_{\omega'}G^{-(A)}(\epsilon_1 - \omega', \epsilon - \omega) \\ & +F^{\dagger(R)}(\epsilon, \epsilon_1)\Delta_{\omega'}F^{(A)}(\epsilon_1 - \omega', \epsilon - \omega)] \\ & +[G^{+(R)}(\epsilon, \epsilon_1)F^{(A)}(\epsilon_1 - \omega', \epsilon - \omega) + F^{\dagger(R)}(\epsilon, \epsilon_1)G^{-(A)}(\epsilon_1 - \omega', \epsilon - \omega)] \\ & \times[e\mathbf{A}_{\omega'} \cdot \mathbf{p}/m - e\varphi_{\omega'}]\}[\tanh\frac{\epsilon_1}{2T} - \tanh\frac{\epsilon_1 - \omega'}{2T}] \ ,\end{aligned} \tag{2.16}$$

where, $G^{-(R,A)}$ and $F^{\dagger(R,A)}$ are formally defined by

$$\begin{aligned}G^{-}(\mathbf{p}, \mathbf{p} - \mathbf{k}; \epsilon, \epsilon - \omega) = G^{0-}(\mathbf{p}; \epsilon) + G^{0-}(\mathbf{p}; \epsilon)\Delta_{\omega'}(\mathbf{k}')G^{0-}(\mathbf{p} - \mathbf{k}'; \epsilon - \omega')\Delta^*_{\omega''}(\mathbf{k}'') \\ \times G^{0-}(\mathbf{p} - \mathbf{k}' - \mathbf{k}''; \epsilon - \omega' - \omega'') + \dots \ ,\end{aligned} \tag{2.17}$$

and

$$F^{\dagger}(\mathbf{p},\mathbf{p}-\mathbf{k};\epsilon,\epsilon-\omega) = -\{G^{0+}(\mathbf{p};\epsilon)\Delta^*_{\omega'}(\mathbf{k}')G^{0-}(\mathbf{p}-\mathbf{k}';\epsilon-\omega') + \ldots\}\ , \tag{2.18}$$

in which the substitution $\epsilon \to \epsilon \pm i0^+$ for the retarded (advanced) Green function should be made. Here $G^{0\pm}(\mathbf{p};\epsilon) = [\epsilon \pm \xi_{\mathbf{p}}]^{-1}$ and all ω^i are real. The functions G^+ and F are obtained from G^- and $F^\dagger$ by changing the sign of ξ in $G^{0\pm}$. Note that G^+ and F are introduced only for simplicity of notation.

B. Normal part

The normal part of Δ^* can be written as

$$\Delta^{*,N}_{\alpha\beta}(\mathbf{R},\mathbf{k};\omega) = \int \frac{d\mathbf{k}'}{(2\pi)^2} V(\mathbf{k}-\mathbf{k}')[T\sum_{\epsilon_n\geq 0} F^{\dagger(R)}_{\alpha\beta}(\mathbf{R},\mathbf{k}';\epsilon_n+\omega,\epsilon_n) + T\sum_{\epsilon_n\leq 0} F^{\dagger(A)}_{\alpha\beta}(\mathbf{R},\mathbf{k}';\epsilon_n,\epsilon_n-\omega)]\ . \tag{2.19}$$

The evaluation of the normal part can be done by expanding the expressions in powers of the order parameter. We write for $F^{\dagger(R,A)}$ and $G^{(R,A)}$ up to terms of the third and second order in Δ, respectively, so that

$$\begin{aligned} F^{\dagger(R,A)}_{\alpha\beta}(\mathbf{x},\mathbf{x}';\epsilon,\epsilon-\omega) &= F^{\dagger(R,A)}_{I1,\alpha\beta}(\mathbf{x},\mathbf{x}';\epsilon,\epsilon-\omega) + F^{\dagger(R,A)}_{I2,\alpha\beta}(\mathbf{x},\mathbf{x}';\epsilon,\epsilon-\omega) \\ &\quad + F^{\dagger(R,A)}_{II,\alpha\beta}(\mathbf{x},\mathbf{x}';\epsilon,\epsilon-\omega)\ , \end{aligned} \tag{2.20}$$

where

$$\begin{aligned} F^{\dagger(R,A)}_{I1,\alpha\beta}(\mathbf{x},\mathbf{x}';\epsilon,\epsilon-\omega) &= \int d\mathbf{x}_1 d\mathbf{x}_2 G^{0(R,A)}_{\mu\alpha}(\mathbf{x}_1,\mathbf{x};-\epsilon) e^{-ie\mathbf{A}_{\omega'}(\mathbf{x}_1)\cdot(\mathbf{x}_1-\mathbf{x})}\Delta^*_{\mu\nu}(\mathbf{x}_1,\mathbf{x}_2;\omega'') \\ &\quad \times G^{0(R,A)}_{\nu\beta}(\mathbf{x}_2,\mathbf{x}';\epsilon-\omega) e^{-ie\mathbf{A}_{\omega'''}(\mathbf{x}_2)\cdot(\mathbf{x}_2-\mathbf{x}')}\ , \end{aligned} \tag{2.21}$$

$$\begin{aligned} F^{\dagger(R,A)}_{I2,\alpha\beta}(\mathbf{x},\mathbf{x}';\epsilon,\epsilon-\omega) &= \int d\mathbf{x}_1 d\mathbf{x}_2 d\mathbf{x}_3 G^{0(R,A)}_{\mu\alpha}(\mathbf{x}_1,\mathbf{x};-\epsilon)\Delta^*_{\mu\nu}(\mathbf{x}_1,\mathbf{x}_2;\omega') G^{0(R,A)}_{\nu\rho}(\mathbf{x}_2,\mathbf{x}_3;\epsilon-\omega') \\ &\quad \times[-e\varphi_{\omega''}(\mathbf{x}_3)] G^{0(R,A)}_{\rho\beta}(\mathbf{x}_3,\mathbf{x}';\epsilon-\omega) \\ &\quad + \int d\mathbf{x}_1 d\mathbf{x}_2 d\mathbf{x}_3 G^{0(R,A)}_{\mu\rho}(\mathbf{x}_1,\mathbf{x}_3;-(\epsilon-\omega'))[-e\varphi_{\omega'}(\mathbf{x}_3)] G^{0(R,A)}_{\rho\alpha}(\mathbf{x}_3,\mathbf{x};-\epsilon) \\ &\quad \times \Delta^*_{\mu\nu}(\mathbf{x}_1,\mathbf{x}_2;\omega'') G^{0(R,A)}_{\nu\beta}(\mathbf{x}_2,\mathbf{x}';\epsilon-\omega)\ , \end{aligned} \tag{2.22}$$

$$\begin{aligned} F^{\dagger(R,A)}_{II,\alpha\beta}(\mathbf{x},\mathbf{x}';\epsilon,\epsilon-\omega) &= -\int d\mathbf{x}_1 d\mathbf{x}_2 d\mathbf{x}_3 d\mathbf{x}_4 d\mathbf{x}_5 d\mathbf{x}_6 G^{0(R,A)}_{\mu\alpha}(\mathbf{x}_1,\mathbf{x};-\epsilon)\Delta^*_{\mu\nu}(\mathbf{x}_1,\mathbf{x}_2;\omega') \\ &\quad \times G^{0(R,A)}_{\nu\lambda}(\mathbf{x}_2,\mathbf{x}_3;\epsilon-\omega')\Delta_{\lambda\sigma}(\mathbf{x}_3,\mathbf{x}_4;\omega'') G^{0(R,A)}_{\rho\sigma}(\mathbf{x}_5,\mathbf{x}_4;-(\epsilon-\omega'-\omega'')) \\ &\quad \times \Delta^*_{\rho\tau}(\mathbf{x}_5,\mathbf{x}_6;\omega''') G^{0(R,A)}_{\tau\beta}(\mathbf{x}_6,\mathbf{x}';\epsilon-\omega)\ . \end{aligned} \tag{2.23}$$

Here the summation over the imaginary frequency ω^i with the constraint $\sum_i \omega^i = \omega$ is implied. To write down the above expression for $F^{\dagger(R,A)}$, we have expanded $\tilde{G}^0$ to the first order in the scalar potential $-e\varphi$, and separate this expansion term out explicitly. As far as the dependence of $\tilde{G}^0$ on the magnetic field is concerned, the quasiclassical phase approximation

can be used to write it in the form $G^0(\mathbf{x}, \mathbf{x}'; \epsilon) \exp[-ie\mathbf{A}_\omega \cdot (\mathbf{x} - \mathbf{x}')]$. Accordingly, the gap function can also be written as a sum of three parts

$$\Delta^{*,N}_{\alpha\beta}(\mathbf{R}, \mathbf{k}; \omega) = \Delta^{*,N}_{I1,\alpha\beta}(\mathbf{R}, \mathbf{k}; \omega) + \Delta^{*,N}_{I2,\alpha\beta}(\mathbf{R}, \mathbf{k}; \omega) + \Delta^{*,N}_{II,\alpha\beta}(\mathbf{R}, \mathbf{k}; \omega) \ . \tag{2.24}$$

The remaining task involves the evaluation of the average over an ensemble of randomly distributed configurations. As an approximation, Δ^* is regarded as very nearly independent of impurity configurations. We assume that impurities density n_i are randomly distributed and their spins are arbitrarily oriented so that there is no correlation among them.[14] Using the Born approximation, we can show that the impurity averaged zero-field normal-state Green function takes the following form

$$\langle G^0_{\alpha\beta}(\mathbf{x}, \mathbf{x}'; \epsilon_n) \rangle = \frac{1}{(2\pi)^2} \int d\mathbf{k} e^{i\mathbf{k}(\mathbf{x}-\mathbf{x}')} \frac{\delta_{\alpha\beta}}{i\epsilon_n \eta_1 - \xi_{\mathbf{k}}}, \tag{2.25}$$

where $\langle \ldots \rangle$ denotes the average over the impurity configuration, $\xi_{\mathbf{k}} = \frac{\mathbf{k}^2}{2m} - \mu$ is the kinetic energy, and $\eta_1 = 1 + (2\tau_1|\epsilon_n|)^{-1}$ with the scattering time τ_1 given by

$$\frac{1}{\tau_1} = 2\pi n_i N(0)[|U_1|^2 + \frac{1}{4} S(S+1)|U_2|^2] \ . \tag{2.26}$$

Here n_i is the impurity density and $N(0)$ is the density of states at the Fermi surface per spin. The evaluation of the product of Green functions can be conveniently performed based on the diagrammatic rule.[15,16] If there are two Green functions connected by an s-wave order parameter or s-channel two-body interaction, these two Green functions might be called directly connected and we should attach a vertex renormalization factor:

$$\begin{aligned} \eta^{(R,A)}(\epsilon) &= [1 - a^{(R,A)}(0)]^{-1} \\ &\approx \frac{\epsilon \pm \frac{i}{2\tau_1}}{\epsilon \pm \frac{i}{\tau_s}} \mp \frac{\frac{i\omega}{\tau_2}}{(2\epsilon \pm \frac{2i}{\tau_s})^2} \ , \end{aligned} \tag{2.27}$$

where

$$a^{(R)}(0) = \frac{1}{2\pi N(0)\tau_2} \int \frac{d\mathbf{p}}{(2\pi)^2} G^{0(R)}(\mathbf{p}, -(\epsilon - i\omega)) G^{0(R)}(\mathbf{p}, \epsilon) \ , \tag{2.28}$$

and

$$a^{(A)}(0) = \frac{1}{2\pi N(0)\tau_2} \int \frac{d\mathbf{p}}{(2\pi)^2} G^{0(A)}(\mathbf{p}, -\epsilon) G^{0(A)}(\mathbf{p}, \epsilon + i\omega) \ . \tag{2.29}$$

Here

$$\frac{1}{\tau_2} = 2\pi n_i N(0)[|U_1|^2 - \frac{1}{4} S(S+1)|U_2|^2] \ , \tag{2.30}$$

and the spin-flip scattering rate is defined as $2\tau_s^{-1} = \tau_1^{-1} - \tau_2^{-1}$. If the two Green functions are connected by a d-wave order parameter or d-channel two-body interaction, they might be called not directly connected and we have no vertex correction. For the average of the product of more than two Green functions, an impurity line can also appear across the box

of a diagram. Because of their zero contribution, diagrams with more than one impurity line across the box should not be included. This impurity averaging technique was first used by Abrikosov and Gorkov[14] for conventional s-wave superconductors. Recent experimental measurements by Bernhard *et al.*[17] on various types of $YBa_2(Cu_{1-x}Zn_x)_3O_{7-\delta}$ samples have shown that the depression of T_c by Zn doping can be fitted well with the Abrikosov-Gorkov theory applied to the d-wave superconductivity.

Now we give a derivation for the gap function from $T\sum_{\epsilon_n\geq 0} F^{\dagger(R)}(\epsilon_n+\omega,\epsilon_n)$. The contribution from $T\sum_{\epsilon_n\leq 0} F^{\dagger(R)}(\epsilon_n,\epsilon_n-\omega)$ can be obtained by merely changing all explicit i to $-i$ and ω to $-\omega$, which gives the same result. In addition, one can easily see that $\Delta^{*,N}_{I2,\alpha\beta}(\mathbf{R},\mathbf{k};\omega)=0$ since the contribution from the two terms given by Eq. (2.22) cancelled with each other. Therefore, we obtain

$$\begin{aligned}\Delta^{*,N(R)}_{I1,\alpha\beta}(\mathbf{R},\mathbf{k};\omega) = T\sum_{\epsilon_n\geq 0}\int\frac{d\mathbf{k}'}{(2\pi)^2}V(\mathbf{k}-\mathbf{k}')\int d\mathbf{r}e^{-i\mathbf{k}'\cdot\mathbf{r}}\int\int d\mathbf{R}'d\mathbf{r}'\\ \times\langle G^{0(R)}_{\lambda\alpha}(\mathbf{R}'+\frac{\mathbf{r}'}{2},\mathbf{R}+\frac{\mathbf{r}}{2};-(\epsilon_n+\omega))g_{\lambda\mu}G^{0(R)}_{\mu\beta}(\mathbf{R}'-\frac{\mathbf{r}'}{2},\mathbf{R}-\frac{\mathbf{r}}{2};\epsilon_n)\rangle\\ \times e^{i(\mathbf{R}'-\mathbf{R})\cdot\mathbf{\Pi}}\int\frac{d\mathbf{k}''}{(2\pi)^2}e^{i\mathbf{k}''\cdot\mathbf{r}'}\Delta^*_\omega(\mathbf{R},\mathbf{k}'')\ ,\end{aligned}\tag{2.31}$$

and

$$\begin{aligned}\Delta^{*,N(R)}_{II,\alpha\beta}(\mathbf{R},\mathbf{k};\omega) = T\sum_{\epsilon_n\geq 0}\int\frac{d\mathbf{k}'}{(2\pi)^2}V(\mathbf{k}-\mathbf{k}')\int d\mathbf{r}e^{-i\mathbf{k}'\cdot\mathbf{r}}\int d\mathbf{R}'d\mathbf{r}'d\mathbf{R}_1d\mathbf{r}_1d\mathbf{R}_2d\mathbf{r}_2\\ \times\langle G^{0(R)}_{\lambda\alpha}(\mathbf{R}'+\frac{\mathbf{r}'}{2},\mathbf{R}+\frac{\mathbf{r}}{2},-\epsilon_n)g_{\lambda\mu}G^{0(R)}_{\mu\nu}(\mathbf{R}'-\frac{\mathbf{r}'}{2},\mathbf{R}_1+\frac{\mathbf{r}_1}{2},\epsilon_n)\\ \times g_{\nu\rho}G^{0(R)}_{\kappa\rho}(\mathbf{R}_2+\frac{\mathbf{r}_2}{2},\mathbf{R}_1-\frac{\mathbf{r}_1}{2},-\epsilon_n)g_{\kappa\sigma}G^{0(R)}_{\sigma\beta}(\mathbf{R}_2-\frac{\mathbf{r}_2}{2},\mathbf{R}-\frac{\mathbf{r}}{2},\epsilon_n)\rangle\\ \times\int\frac{d\mathbf{k}''d\mathbf{k}_1d\mathbf{k}_2}{(2\pi)^6}\Delta^*_{\omega'}(\mathbf{R},\mathbf{k}')\Delta_{\omega''}(\mathbf{R},\mathbf{k}_1)\Delta^*_{\omega'''}(\mathbf{R},\mathbf{k}_2)e^{i(\mathbf{k}''\cdot\mathbf{r}'+\mathbf{k}_1\cdot\mathbf{r}_1+\mathbf{k}_1\cdot\mathbf{r}_2)}\ .\end{aligned}\tag{2.32}$$

Here $\mathbf{\Pi}=-i\nabla_{\mathbf{R}}-2e\mathbf{A}_{\omega'}(\mathbf{R})$ and we have assumed the slow variation of the vector potential.

Since $G^0_{\alpha\beta}=G^0\delta_{\alpha\beta}$ is diagonal in the spin space, we can express $\Delta^{*,N(R)}_{I1,\alpha\beta}=\Delta^{*,N(R)}_{I1}g_{\alpha\beta}$ with

$$\Delta^{*,N(R)}_{I1}(\mathbf{R},\mathbf{k};\omega)=\Delta^{*,N(R)}_{I1c}(\mathbf{R},\mathbf{k};\omega)+\Delta^{*,N(R)}_{I1g}(\mathbf{R},\mathbf{k};\omega)\ ,\tag{2.33}$$

where

$$\begin{aligned}\Delta^{*,N(R)}_{I1c}(\mathbf{R},\mathbf{k}) = T\sum_{\epsilon_n\geq 0}\int\frac{d\mathbf{k}'}{(2\pi)^2}V(\mathbf{k}-\mathbf{k}')\int d\mathbf{r}e^{-i\mathbf{k}'\cdot\mathbf{r}}\int\int d\mathbf{R}'d\mathbf{r}'\\ \times\langle G^{0(R)}_{\lambda\alpha}(\mathbf{R}'+\frac{\mathbf{r}'}{2},\mathbf{R}+\frac{\mathbf{r}}{2};-(\epsilon_n+\omega))g_{\lambda\mu}G^{0(R)}_{\mu\beta}(\mathbf{R}'-\frac{\mathbf{r}'}{2},\mathbf{R}-\frac{\mathbf{r}}{2};\epsilon_n)\rangle\\ \times\int\frac{d\mathbf{k}''}{(2\pi)^2}e^{i\mathbf{k}''\cdot\mathbf{r}'}\Delta^*_\omega(\mathbf{R},\mathbf{k}'')\ ,\end{aligned}\tag{2.34}$$

and

$$\Delta^*_{Ig}(\mathbf{R},\mathbf{k}) = -\frac{1}{2}T\sum_{\epsilon_n\ge 0}\int\frac{d\mathbf{k}'}{(2\pi)^2}V(\mathbf{k}-\mathbf{k}')\int d\mathbf{r}e^{-i\mathbf{k}'\cdot\mathbf{r}}\int\int d\mathbf{R}'d\mathbf{r}'$$
$$\times\langle G^{0(R)}_{\lambda\alpha}(\mathbf{R}'+\frac{\mathbf{r}'}{2},\mathbf{R}+\frac{\mathbf{r}}{2};-(\epsilon_n+\omega))g_{\lambda\mu}G^{0(R)}_{\mu\beta}(\mathbf{R}'-\frac{\mathbf{r}'}{2},\mathbf{R}-\frac{\mathbf{r}}{2};\epsilon_n)\rangle$$
$$\times[(\mathbf{R}'-\mathbf{R})\cdot\mathbf{\Pi}]^2\int\frac{d\mathbf{k}''}{(2\pi)^2}e^{i\mathbf{k}''\cdot\mathbf{r}'}\Delta^*_\omega(\mathbf{R},\mathbf{k}'')\ , \tag{2.35}$$

Similarly, $\Delta^{*,N(R)}_{II,\alpha\beta} = \Delta^{*,N(R)}_{II}g_{\alpha\beta}$.

Using the diagrammatic rule mentioned above, we calculate $\Delta^{*,N(R)}_{Ic}$, which in general has four terms for the mixed s and d wave superconductors, $\Delta^{*,N(R)}_{Ic} = \sum_{i=1}^4\Delta^{*,N(R)}_{Ic,i}$. It is easy to show that

$$\begin{aligned}\Delta^{*,N(R)}_{Ic,1}(\mathbf{R},\mathbf{k};\omega) &= T\sum_{\epsilon_n\ge 0}\int\frac{d\mathbf{p}}{(2\pi)^2}\eta^{(R)}V_sG^{0(R)}(-\mathbf{p};-(\epsilon_n+\omega))G^{0(R)}(\mathbf{p};\epsilon_n)\Delta^*_s(\mathbf{R},\omega)\\ &= \frac{V_sN(0)}{2}\{[\ln\frac{2e^\gamma\omega_D}{\pi T}+\psi(\frac{1}{2})-\psi(\frac{1}{2}+\frac{\rho_s}{2})]+\frac{i\omega}{4\pi T}\psi'(\frac{1}{2}+\frac{\rho_s}{2})\}\\ &\quad\times\Delta^*_s(\mathbf{R},\omega)\ ,\end{aligned} \tag{2.36a}$$

$$\begin{aligned}\Delta^{*,N(R)}_{Ic,2}(\mathbf{R},\mathbf{k};\omega) &= T\sum_{\epsilon_n\ge 0}\int\frac{d\mathbf{p}}{(2\pi)^2}\eta^{(R)}V_sG^{0(R)}(-\mathbf{p};-(\epsilon_n+\omega))G^{0(R)}(\mathbf{p};\epsilon_n)\Delta^*_d(\mathbf{R},\omega)(\hat{p}_x^2-\hat{p}_y^2)\\ &= 0\ ,\end{aligned} \tag{2.36b}$$

$$\begin{aligned}\Delta^{*,N(R)}_{Ic,3}(\mathbf{R},\mathbf{k};\omega) &= T\sum_{\epsilon_n\ge 0}\int\frac{d\mathbf{p}}{(2\pi)^2}\eta^{(R)}V_d(\hat{k}_x^2-\hat{k}_y^2)(\hat{p}_x^2-\hat{p}_y^2)G^{0(R)}(-\mathbf{p};-(\epsilon_n+\omega))G^{0(R)}(\mathbf{p};\epsilon_n)\\ &\quad\times\Delta^*_s(\mathbf{R},\omega)\\ &= 0\ ,\end{aligned} \tag{2.36c}$$

$$\begin{aligned}\Delta^{*,N(R)}_{Ic,4}(\mathbf{R},\mathbf{k};\omega) &= T\sum_{\epsilon_n\ge 0}\int\frac{d\mathbf{p}}{(2\pi)^2}V_d(\hat{k}_x^2-\hat{k}_y^2)(\hat{p}_x^2-\hat{p}_y^2)G^{0(R)}(-\mathbf{p};-(\epsilon_n+\omega))G^{0(R)}(\mathbf{p};\epsilon_n)\\ &\quad\times\Delta^*_d(\mathbf{R},\omega)(\hat{p}_x^2-\hat{p}_y^2)\\ &= \frac{V_dN(0)}{4}\{[\ln\frac{2e^\gamma\omega_D}{\pi T}+\psi(\frac{1}{2})-\psi(\frac{1}{2}+\frac{\rho_1}{2})]+\frac{i\omega}{4\pi T}\psi'(\frac{1}{2}+\frac{\rho_1}{2})\}\\ &\quad\times\Delta^*_d(\mathbf{R},\omega)(\hat{k}_x^2-\hat{k}_y^2)\ ,\end{aligned} \tag{2.36d}$$

where γ is the Euler constant, ω_0 is the cut-off frequency, $\rho_s = 1/\pi T\tau_s$, $\rho_1 = 1/2\pi\tau_1$, $\psi'(x)$ is the derivative of the digamma function $\psi(x)$.

As for the results of $\Delta^{*,N(R)}_{Ig} = \sum_{i=1}^4\Delta^{*,N(R)}_{Ig,i}$ and $\Delta^{*,N(R)}_{II} = \sum_{i=1}^{16}\Delta^{*,N(R)}_{II,i}$, the second term in Eq. (2.27) can be dropped since it gives a very small higher order correction due to the fact that the gap function usually has a temporal variation over a time scale very long compared to the range of the Green function, that is, $\omega\tau_1 \ll 1$. Therefore, the details to evaluate them are the same as the static case.[15,16] Here we just give the results

$$\Delta_{I1g}^{*,N(R)}(\mathbf{R},\mathbf{k};\omega) = -\frac{V_sN(0)}{8}(\frac{v_F}{\pi T})^2[\chi_{2,1}\mathbf{\Pi}^2\Delta_s^*(\mathbf{R};\omega') + \frac{1}{2}\chi_{1,2}(\Pi_x^2-\Pi_y^2)\Delta_d^*(\mathbf{R};\omega')]$$
$$-\frac{V_d(\hat{k}_x^2-\hat{k}_y^2)N(0)}{16}(\frac{v_F}{\pi T})^2[\chi_{1,2}(\Pi_x^2-\Pi_y^2)\Delta_s^*(\mathbf{R};\omega')$$
$$+\chi_{0,3}\mathbf{\Pi}^2\Delta_d^*(\mathbf{R};\omega')] \tag{2.37}$$

$$\Delta_{II}^{*,N(R)}(\mathbf{R},\mathbf{k};\omega) = -\frac{V_sN(0)}{2(\pi T)^2}\{[\chi_{3,0}-\rho_s\chi_{4,0}]\Delta_s^*(\mathbf{R};\omega')\Delta_s(\mathbf{R};\omega'')\Delta_s^*(\mathbf{R};\omega''')$$
$$+[\chi_{2,1}-\frac{\rho_1}{2}\chi_{2,2}]\Delta_d^*(\mathbf{R};\omega')\Delta_d(\mathbf{R};\omega'')\Delta_s^*(\mathbf{R};\omega''')$$
$$+\frac{1}{2}\chi_{2,1}\Delta_d^*(\mathbf{R};\omega')\Delta_s(\mathbf{R};\omega'')\Delta_d^*(\mathbf{R};\omega''']$$
$$-\frac{V_d(\hat{k}_x^2-\hat{k}_y^2)N(0)}{4(\pi T)^2}\{\chi_{2,1}\Delta_s^*(\mathbf{R};\omega')\Delta_d(\mathbf{R};\omega'')\Delta_s^*(\mathbf{R};\omega''')$$
$$+[\chi_{2,1}-\frac{\rho_1}{2}\chi_{2,2}]\Delta_d^*(\mathbf{R};\omega')\Delta_s(\mathbf{R};\omega'')\Delta_s^*(\mathbf{R};\omega''')$$
$$+\frac{3}{4}\chi_{0,3}\Delta_d^*(\mathbf{R};\omega')\Delta_d(\mathbf{R};\omega'')\Delta_d^*(\mathbf{R};\omega''']\}\ , \tag{2.38}$$

where v_F is the Fermi velocity, and $\chi_{m,m'}$ is a function defined as

$$\chi_{m,m'} = \sum_{n\geq 0}\frac{1}{(2n+1+\rho_s)^m(2n+1+\rho_1)^{m'}}\ . \tag{2.39}$$

C. Anomalous part

The anomalous part contains integrals of the products of the retarded and advanced Green functions, and is therefore sensitive to the details of the spectrum. Following Ref.[7], we summarize here the diagrammatic rule for the evaluation of this part. In each diagram, the solid (electron) lines forming the upper part of the diagram correspond to the retarded Green function $G^{0(R)}(\mathbf{p};\epsilon) = [\epsilon-\xi_\mathbf{p}+i/2\tau_1]^{-1}$ for those lines with arrows to the right and to $G^{0(R)}(-\mathbf{p};-\epsilon) = [-\epsilon-\xi_{-\mathbf{p}}-i/2\tau_1]^{-1}$ for those with arrows to the left. The solid lines in the lower part of the diagram correspond to the advanced Green function $G^{0(A)}(\mathbf{p};\epsilon) = [\epsilon-\xi_\mathbf{p} - i/2\tau_1]^{-1}$ for those lines with arrows to the left and to $G^{0(A)}(-\mathbf{p};-\epsilon) = [-\epsilon-\xi_{-\mathbf{p}}+i/2\tau_1]^{-1}$ for those with arrows to the right. The triangle and the thin wavy line represent the order parameter Δ and the vertex interaction with the electromagnetic field, respectively. The dashed line corresponds to the impurity scattering. If the dashed line encompasses an even number of Δ, a factor $1/2\pi\tau_1N(0)$ should be assigned. If it encompasses an odd number of Δ, a factor $1/2\pi\tau_2N(0)$ should be assigned.

As shown in Fig. 1, the staircase which is the summation of the ladder diagrams, has a singular value. We denote it by $I(\omega,\mathbf{k})$, which satisfies a ladder-type equation

$$I(\omega,\mathbf{k}) = \frac{1}{2\pi\tau_1N(0)}\{1+\int d\mathbf{p}G^{0(R)}(\mathbf{p};\epsilon)G^{0(A)}(\mathbf{p}-\mathbf{k};\epsilon-\omega)I(\omega,\mathbf{k})\}$$

$$= \frac{1}{2\pi\tau_1 N(0)}\{1 + N(0)\int\int \frac{d\xi d\theta/2\pi}{(\epsilon - \xi + \frac{i}{2\tau_1})(\epsilon - \omega + \mathbf{v}_F\cdot\mathbf{k} - \frac{i}{2\tau_1})} I(\omega,\mathbf{k})\}$$
$$= \frac{1}{2\pi\tau_1 N(0)}\{1 + N(0)\int \frac{(2\pi i)d\theta/2\pi}{\omega - \mathbf{v}_F\cdot\mathbf{k} + \frac{i}{\tau_1}} I(\omega,\mathbf{k})\}\ . \tag{2.40}$$

Under the condition $\omega\tau_1 \ll 1$ and $v_F k\tau_1 \ll 1$, we obtain

$$I(\omega,\mathbf{k}) = \frac{1}{2\pi\tau_1 N(0)} \frac{1}{(-i\omega + Dk^2)\tau_1}\ , \tag{2.41}$$

where $D = v_F^2\tau_1/2$ is the difussion constant for the two dimensional systems. In the real time and coordinate space, I^{-1} is proportional to the operator $\frac{\partial}{\partial t} - D\nabla^2$. It is important to note that the denominator of Eq. (2.41) can be very small if $\omega\tau_1$ and $v_F k\tau_1$ are both small. This fact makes it necessary to sum additionally diagrams containing arbitrary number of staircases $I(\omega,\mathbf{k})$, separated by parts including Δ and Δ^*. Under the assumption $\tau_s\Delta_s \ll 1$ and $\tau_1\Delta_1 \ll 1$, we need only be concerned with those diagrams of the order of Δ^2. These diagrams together lead to the diffusion equation for the vertex parts Γ^+ and Γ^- as shown in Fig. 2.

The kernel Q_1 corresponds to the diagrams shown in Fig. 3 and is given by

$$Q_1(\mathbf{R};\omega) = Q_1^{(a)}(\mathbf{R};\omega) + Q_1^{(b)}(\mathbf{R};\omega) + Q_1^{(c)}(\mathbf{R};\omega) \tag{2.42}$$

with

$$Q_1^{(a)}(\mathbf{R};\omega) = -\int \frac{d\mathbf{p}}{(2\pi)^2}[G^{0(R)}(\mathbf{p};\epsilon)]^2 G^{0(R)}(-\mathbf{p};-\epsilon)G^{0(A)}(\mathbf{p};\epsilon)$$
$$\times\tilde{\Delta}_{\omega'}(\mathbf{R},\mathbf{p})\tilde{\Delta}^*_{\omega}(\mathbf{R},\mathbf{p})\ , \tag{2.43}$$

$$Q_1^{(b)}(\mathbf{R};\omega) = -\frac{1}{2\pi\tau_2 N(0)}\int \frac{d\mathbf{p}}{(2\pi)^2} G^{0(R)}(\mathbf{p};\epsilon)G^{0(R)}(-\mathbf{p};-\epsilon)G^{0(A)}(\mathbf{p};\epsilon)\tilde{\Delta}_{\omega'}(\mathbf{R},\mathbf{p})$$
$$\times\int \frac{d\mathbf{p}'}{(2\pi)^2} G^{0(R)}(\mathbf{p}';\epsilon)G^{0(R)}(-\mathbf{p}';-\epsilon)G^{0(A)}(\mathbf{p}';\epsilon)\tilde{\Delta}^*_{\omega}(\mathbf{R},\mathbf{p}')\ , \tag{2.44}$$

$$Q_1^{(c)}(\mathbf{R};\omega) = -\frac{1}{2\pi\tau_1 N(0)}\int \frac{d\mathbf{p}'}{(2\pi)^2}[G^{0(R)}(\mathbf{p}';\epsilon)]^2 G^{0(A)}(\mathbf{p}';\epsilon)$$
$$\times\int \frac{d\mathbf{p}}{(2\pi)^2}[G^{0(R)}(\mathbf{p};\epsilon)]^2 G^{0(R)}(-\mathbf{p};-\epsilon)\tilde{\Delta}_{\omega'}(\mathbf{R},\mathbf{p})\tilde{\Delta}^*_{\omega}(\mathbf{R},\mathbf{p})\ . \tag{2.45}$$

Here for simplicity of notation, the vortex renormalization factor is included in the order parameter. For the d-wave component, it is unrenormalized, i.e., $\tilde{\Delta}_d = \Delta_d$; while for the s-wave component, $\tilde{\Delta}_s = \eta^{(R,A)}\Delta_s$ or Δ_s depends on whether the vertex connects only retarded and only advanced Green functions or it connects a retarded and an advanced Green function.

In view of the specific form of the s-wave and d-wave pairing states, we see that the contribution from the s-wave and d-wave component is decoupled. Therefore, we obtain

$$
\begin{aligned}
Q_1(\mathbf{R};\omega) &= Q_{1,s}(\mathbf{R};\omega) + Q_{1,d}(\mathbf{R};\omega) \\
&= -\frac{\pi i\tau_1^2 N(0)}{\epsilon + \frac{i}{\tau_s}}\Delta_s(\mathbf{R};\omega')\Delta_s^*(\mathbf{R};\omega) - \frac{\pi i\tau_1^2 N(0)}{2(\epsilon + \frac{i}{2\tau_1})}\Delta_d(\mathbf{R};\omega')\Delta_d^*(\mathbf{R};\omega)\ . \qquad (2.46)
\end{aligned}
$$

We can find Q_2 from Q_1 by merely replacing all explicit i's by $-i$'s

$$
Q_2(\mathbf{R};\omega) = \frac{\pi i\tau_1^2 N(0)}{\epsilon - \frac{i}{\tau_s}}\Delta_s(\mathbf{R};\omega')\Delta_s^*(\mathbf{R};\omega) + \frac{\pi i\tau_1^2 N(0)}{2(\epsilon - \frac{i}{2\tau_1})}\Delta_d(\mathbf{R};\omega')\Delta_d^*(\mathbf{R};\omega) \qquad (2.47)
$$

The diagram shown in Fig. 4 leads to

$$
Q_3(\mathbf{R};\omega) = Q_3^{(a)}(\mathbf{R};\omega) + Q_3^{(b)}(\mathbf{R};\omega) + Q_3^{(c)}(\mathbf{R};\omega)\ , \qquad (2.48)
$$

with

$$
\begin{aligned}
Q_3^{(a)}(\mathbf{R};\omega) = \int \frac{d\mathbf{p}}{(2\pi)^2} & G^{0(R)}(\mathbf{p};\epsilon)G^{0(R)}(-\mathbf{p};-\epsilon)G^{0(A)}(\mathbf{p};\epsilon)G^{0(A)}(-\mathbf{p};-\epsilon) \\
& \times\tilde{\Delta}_{\omega'}(\mathbf{R},\mathbf{p})\tilde{\Delta}_{\omega}^*(\mathbf{R},\mathbf{p})\ , \qquad (2.49)
\end{aligned}
$$

$$
\begin{aligned}
Q_3^{(b)}(\mathbf{R};\omega) = \frac{1}{2\pi\tau_2 N(0)} & \int \frac{d\mathbf{p}}{(2\pi)^2}G^{0(R)}(\mathbf{p};\epsilon)G^{0(R)}(-\mathbf{p};-\epsilon)G^{0(A)}(-\mathbf{p};-\epsilon)\tilde{\Delta}_{\omega'}(\mathbf{R},\mathbf{p}) \\
& \times\int \frac{d\mathbf{p}'}{(2\pi)^2}G^{0(R)}(\mathbf{p}';\epsilon)G^{0(A)}(\mathbf{p}';\epsilon)G^{0(A)}(-\mathbf{p}';-\epsilon)\tilde{\Delta}_{\omega}^*(\mathbf{R},\mathbf{p}')\ , \qquad (2.50)
\end{aligned}
$$

$$
\begin{aligned}
Q_3^{(c)}(\mathbf{R};\omega) = \frac{1}{2\pi\tau_2 N(0)} & \int \frac{d\mathbf{p}}{(2\pi)^2}G^{0(R)}(\mathbf{p};\epsilon)G^{0(R)}(-\mathbf{p};-\epsilon)G^{0(A)}(\mathbf{p};\epsilon)\tilde{\Delta}_{\omega'}(\mathbf{R},\mathbf{p}) \\
& \times\int \frac{d\mathbf{p}'}{(2\pi)^2}G^{0(R)}(-\mathbf{p}';-\epsilon)G^{0(A)}(\mathbf{p}';\epsilon)G^{0(A)}(-\mathbf{p}';-\epsilon)\tilde{\Delta}_{\omega}^*(\mathbf{R},\mathbf{p}')\ . \qquad (2.51)
\end{aligned}
$$

The algebra gives

$$
Q_3(\mathbf{R};\omega) = \frac{2\pi N(0)\tau_1^2/\tau_s}{\epsilon^2 + \tau_s^{-2}}\Delta_s(\mathbf{R};\omega_1)\Delta_s^*(\mathbf{R};\omega_2) + \frac{2\pi N(0)\tau_1^2}{2[\epsilon^2 + (2\tau_1)^{-2}]}\Delta_d(\mathbf{R};\omega_1)\Delta_d^*(\mathbf{R};\omega_2)\ . \quad (2.52)
$$

From the results of $Q_{1,2,3}$ given by Eqs. (2.46), (2.47), and (2.52), it is not difficult to prove the relation

$$
Q_3(\mathbf{R};\omega) = -(Q_1(\mathbf{R};\omega) + Q_2(\mathbf{R};\omega))\ . \qquad (2.53)
$$

In addition, the other three separate terms are as follows:

$$
\begin{aligned}
S_1(\mathbf{R};\omega) = -\int & \frac{d\mathbf{p}}{(2\pi)^2}G^{0(R)}(\mathbf{p};\epsilon)G^{0(R)}(-\mathbf{p};-\epsilon)G^{0(A)}(\mathbf{p};\epsilon) \\
& \times(\tanh\frac{\epsilon}{2T} - \tanh\frac{\epsilon-\omega_2}{2T})\tilde{\Delta}_{\omega_1}(\mathbf{R},\mathbf{p})\Delta_{\omega_2}^*(\mathbf{R},\mathbf{p})\ , \\
= & \frac{1}{2T}\cosh^{-2}\frac{\epsilon}{2T}[\frac{\pi\tau_1 N(0)}{\epsilon + i/\tau_s}\Delta_s(\mathbf{R};\omega_1)\omega_2\Delta_s^*(\mathbf{R};\omega_2) \\
& +\frac{\pi\tau_1 N(0)}{2(\epsilon + i/2\tau_1)}\Delta_d(\mathbf{R};\omega_1)\omega_2\Delta_d^*(\mathbf{R};\omega_2)\ , \qquad (2.54)
\end{aligned}
$$

$$
\begin{aligned}
S_2(\mathbf{R};\omega) = & -\int \frac{d\mathbf{p}}{(2\pi)^2} G^{0(R)}(\mathbf{p};\epsilon)G^{0(A)}(-\mathbf{p};-\epsilon)G^{0(A)}(\mathbf{p};\epsilon) \\
& \times(\tanh\frac{\epsilon}{2T} - \tanh\frac{\epsilon-\omega_1}{2T})\Delta_{\omega_1}(\mathbf{R},\mathbf{p})\tilde{\Delta}^*_{\omega_2}(\mathbf{R},\mathbf{p}) , \\
= & \frac{1}{2T}\cosh^{-2}\frac{\epsilon}{2T}[\frac{\pi\tau_1 N(0)}{\epsilon - i/\tau_s}\Delta_s(\mathbf{R};\omega_1)\omega_2\Delta^*_s(\mathbf{R};\omega_2) \\
& + \frac{\pi\tau_1 N(0)}{2(\epsilon - i/2\tau_1)}\Delta_d(\mathbf{R};\omega_1)\omega_2\Delta^*_d(\mathbf{R};\omega_2) ,
\end{aligned} \tag{2.55}
$$

and

$$
\begin{aligned}
S_3(\mathbf{R};\omega) = & \int \frac{d\mathbf{p}}{(2\pi)^2}(\tanh\frac{\epsilon}{2T} - \tanh\frac{\epsilon-\omega}{2T})G^{0(R)}(\mathbf{p};\epsilon)G^{0(A)}(\mathbf{p};\epsilon)e\varphi_\omega(\mathbf{R}) \\
= & \frac{\pi i\tau_1 N(0)}{2T}\cosh^{-2}\frac{\epsilon}{2T}(-i\omega)e\varphi_\omega(\mathbf{R}) .
\end{aligned} \tag{2.56}
$$

Here we have approximated

$$
\tanh\frac{\epsilon}{2T} - \tanh\frac{\epsilon-\omega}{2T} \approx \frac{\omega}{2T}\cosh^{-2}\frac{\epsilon}{2T} , \tag{2.57}
$$

when $\omega \ll T$.

From the results for Q's and S's, we obtain the diffusion equation for Γ^+

$$
\begin{aligned}
(\frac{\partial}{\partial t} - D\nabla^2)\Gamma^+ = & -\frac{1}{2\pi\tau_1^2 N(0)}[(\sum_{i=1}^{3} S_i + \sum_{i=1}^{2}\Gamma^+) - Q_3\Gamma^-] \\
= & \frac{1}{4T\tau_1}\cosh^{-2}\frac{\epsilon}{2T}\{-\frac{i\epsilon}{\epsilon^2+\tau_s^{-2}}\frac{\partial|\Delta_s|^2}{\partial t} - \frac{i\epsilon}{2[\epsilon^2+(2\tau_1)^{-2}]}\frac{\partial|\Delta_d|^2}{\partial t} - 2ie\frac{\partial\varphi}{\partial t}\} \\
& -\frac{\tau_s^{-1}}{\epsilon^2+\tau_s^{-2}}(\Delta_s\frac{\partial\Delta_s^*}{\partial t} - \Delta_s^*\frac{\partial\Delta_s}{\partial t}) - \frac{(2\tau_1)^{-1}}{2[\epsilon^2+(2\tau_1)^{-2}]}(\Delta_d\frac{\partial\Delta_d^*}{\partial t} - \Delta_d^*\frac{\partial\Delta_d}{\partial t}) \\
& -\{\frac{\tau_s^{-1}}{\epsilon^2+\tau_s^{-2}}|\Delta_s|^2 + \frac{(2\tau_1)^{-1}}{2[\epsilon^2+(2\tau_1)^{-2}]}|\Delta_d|^2\}(\Gamma^+ + \Gamma^-) ,
\end{aligned} \tag{2.58a}
$$

where $\Delta_{s,d}$ and φ are functions of $\mathbf{R}$ and t. and $\Gamma^\pm$ are funtions of $\mathbf{R}$, t, and ϵ.

Similary, the diffusion equation for Γ^- is found to be

$$
\begin{aligned}
(\frac{\partial}{\partial t} - D\nabla^2)\Gamma^- = & \frac{1}{4T\tau_1}\cosh^{-2}\frac{\epsilon}{2T}\{\frac{i\epsilon}{\epsilon^2+\tau_s^{-2}}\frac{\partial|\Delta_s|^2}{\partial t} + \frac{i\epsilon}{2[\epsilon^2+(2\tau_1)^{-2}]}\frac{\partial|\Delta_d|^2}{\partial t} - 2ie\frac{\partial\varphi}{\partial t}\} \\
& -\frac{\tau_s^{-1}}{\epsilon^2+\tau_s^{-2}}(\Delta_s\frac{\partial\Delta_s^*}{\partial t} - \Delta_s^*\frac{\partial\Delta_s}{\partial t}) - \frac{(2\tau_1)^{-1}}{2[\epsilon^2+(2\tau_1)^{-2}]}(\Delta_d\frac{\partial\Delta_d^*}{\partial t} - \Delta_d^*\frac{\partial\Delta_d}{\partial t}) \\
& -\{\frac{\tau_s^{-1}}{\epsilon^2+\tau_s^{-2}}|\Delta_s|^2 + \frac{(2\tau_1)^{-1}}{2[\epsilon^2+(2\tau_1)^{-2}]}|\Delta_d|^2\}(\Gamma^+ + \Gamma^-) .
\end{aligned} \tag{2.58b}
$$

These two diffusion equations can be rewritten as

$$
(\frac{\partial}{\partial t} - D\nabla^2)(\Gamma^+ - \Gamma^-) = -\frac{i\cosh^{-2}(\frac{\epsilon}{2T})}{2T\tau_1}\{\frac{\epsilon}{\epsilon^2+\tau_s^{-2}}\frac{\partial|\Delta_s|^2}{\partial t} + \frac{\epsilon}{2[\epsilon^2+(2\tau_1)^{-2}]}\frac{\partial|\Delta_d|^2}{\partial t}\} , \tag{2.59a}
$$

$$
\begin{aligned}
(\frac{\partial}{\partial t} - D\nabla^2)(\Gamma^+ + \Gamma^-) = -\frac{\cosh^{-2}(\frac{\epsilon}{2T})}{2T\tau_1}\{ & \frac{\tau_s^{-1}}{\epsilon^2+\tau_s^{-2}}(\Delta_s\frac{\partial\Delta_s^*}{\partial t} - \Delta_s^*\frac{\partial\Delta_s}{\partial t}) \\
& + \frac{(2\tau_1)^{-1}}{2[\epsilon^2+(2\tau_1)^{-2}]}(\Delta_d\frac{\partial\Delta_d^*}{\partial t} - \Delta_d^*\frac{\partial\Delta_d}{\partial t}) - 2ie\frac{\partial\varphi}{\partial t}\} \\
& - \{\frac{2\tau_s^{-1}}{\epsilon^2+\tau_s^{-2}}|\Delta_s|^2 + \frac{(2\tau_1)^{-1}}{\epsilon^2+(2\tau_1)^{-2}}|\Delta_d|^2\}(\Gamma^+ + \Gamma^-) \ . \qquad (2.59b)
\end{aligned}
$$

With the results of $\Gamma^\pm$, the anomalous part represented in Fig. 5 is given by

$$
\begin{aligned}
\Theta = & \frac{1}{4\pi i}\{-\int d\epsilon \int \frac{d\mathbf{p}}{(2\pi)^2}V(\mathbf{k}-\mathbf{p})G^{0(R)}(-\mathbf{p};-\epsilon)G^{0(R)}(\mathbf{p};\epsilon)G^{0(A)}(\mathbf{p};\epsilon)\tilde{\Delta}^*_{\omega_1}(\mathbf{R},\mathbf{p})\Gamma^+_{\omega_2}(\mathbf{R},\epsilon) \\
& - \int d\epsilon \int \frac{d\mathbf{p}}{(2\pi)^2}V(\mathbf{k}-\mathbf{p})G^{0(R)}(-\mathbf{p};-\epsilon)G^{0(A)}(-\mathbf{p};-\epsilon)G^{0(A)}(\mathbf{p};\epsilon)\tilde{\Delta}^*_{\omega_1}(\mathbf{R},\mathbf{p})\Gamma^-_{\omega_2}(\mathbf{R},\epsilon)\} \\
= & -\frac{\tau_1 N(0)}{4}V_s\Delta_s^*(\mathbf{R};\omega_1)\int d\epsilon[\frac{\epsilon - i\tau_s^{-1}}{\epsilon+\tau_s^{-2}}\Gamma^+_{\omega_2}(\mathbf{R},\epsilon) - \frac{\epsilon + i\tau_s^{-1}}{\epsilon+\tau_s^{-2}}\Gamma^-_{\omega_2}(\mathbf{R},\epsilon)] \\
& - \frac{\tau_1 N(0)}{8}V_d(\hat{k}_x^2 - \hat{k}_y^2)\Delta_d^*(\mathbf{R};\omega_1)\int d\epsilon[\frac{\epsilon - i(2\tau_1)^{-1}}{\epsilon+(2\tau_1)^{-2}}\Gamma^+_{\omega_2}(\mathbf{R},\epsilon) \\
& - \frac{\epsilon + i(2\tau_1)^{-1}}{\epsilon+(2\tau_1)^{-2}}\Gamma^-_{\omega_2}(\mathbf{R},\epsilon)] \ . \qquad (2.60)
\end{aligned}
$$

where Θ is a function of $\mathbf{R}$, ω, and $\mathbf{k}$.

D. TDGL equations for the order parameters

From Eqs. (2.36), (2.37), and (2.38), and (2.60), by performing the inverse Fourier transform and comparing both sides of the gap function for $\hat{\mathbf{k}}$-independent terms and the terms proportional to $\hat{k}_x^2 - \hat{k}_y^2$, we obtain the coupled TDGL equations for the order parameter components:

$$
\begin{aligned}
& -\tfrac{1}{\gamma_s}\tfrac{\partial\Delta_s^*(\mathbf{R},t)}{\partial t} + 2\Phi_s(\mathbf{R},t)\Delta_s^*(\mathbf{R},t) = \\
& 2\alpha_s\Delta_s^*(\mathbf{R},t) + (\tfrac{v_F}{\pi T})^2[\tfrac{1}{2}\chi_{2,1}\mathbf{\Pi}^2\Delta_s^*(\mathbf{R},t) + \tfrac{1}{4}\chi_{1,2}(\Pi_x^2 - \Pi_y^2)\Delta_d^*(\mathbf{R},t)] \\
& + (\tfrac{1}{\pi T})^2\{2(\chi_{3,0} - \rho_s\chi_{4,0})\Delta_s^*(\mathbf{R},t)|\Delta_s(\mathbf{R},t)|^2 \\
& + 2(\chi_{2,1} - \tfrac{\rho_1}{2}\chi_{2,2})|\Delta_d(\mathbf{R},t)|^2\Delta_s^*(\mathbf{R},t) + \chi_{2,1}\Delta_d^{*2}(\mathbf{R},t)\Delta_s(\mathbf{R},t)\} \ , \qquad (2.61)
\end{aligned}
$$

$$
\begin{aligned}
& -\tfrac{1}{\gamma_d}\tfrac{\partial\Delta_d^*(\mathbf{R},t)}{\partial t} + 2\Phi_d(\mathbf{R},t)\Delta_d^*(\mathbf{R},t) = \\
& \alpha_d\Delta_d^*(\mathbf{R},t) + (\tfrac{v_F}{2\pi T})^2[\chi_{0,3}\mathbf{\Pi}^2\Delta_d^*(\mathbf{R},t) + \chi_{1,2}(\Pi_x^2 - \Pi_y^2)\Delta_s^*(\mathbf{R},t)] \\
& + (\tfrac{1}{\pi T})^2\{(\tfrac{3}{4}\chi_{0,3}\Delta_d^*(\mathbf{R},t)|\Delta_d(\mathbf{R},t)|^2 \\
& + 2(\chi_{2,1} - \tfrac{\rho_1}{2}\chi_{2,2})|\Delta_s(\mathbf{R},t)|^2\Delta_d^*(\mathbf{R},t) + \chi_{2,1}\Delta_s^{*2}(\mathbf{R},t)\Delta_d(\mathbf{R},t)\} \ . \qquad (2.62)
\end{aligned}
$$

Here $\gamma_{s,d}$ are two relaxation rates defined by

$$
\gamma_s^{-1} = \frac{1}{2\pi T}\psi'(\frac{1}{2} + \frac{\rho_s}{2}) \ , \qquad (2.63)
$$

and

$$\gamma_d^{-1} = \frac{1}{4\pi T}\psi'(\frac{1}{2} + \frac{\rho_1}{2}) \ . \tag{2.64}$$

Two quantities $\Phi_{s,d}$ are given by

$$\mu_s(\mathbf{R},t) = -\frac{i\tau_1}{4}\int d\epsilon[\frac{\epsilon - i\tau_s^{-1}}{\epsilon^2 + \tau_s^{-2}}\Gamma^+(\mathbf{R},t,\epsilon) - \frac{\epsilon + i\tau_s^{-1}}{\epsilon^2 + \tau_s^{-2}}\Gamma^-(\mathbf{R},t,\epsilon)] \ , \tag{2.65}$$

$$\mu_d(\mathbf{R},t) = -\frac{i\tau_1}{8}\int d\epsilon[\frac{\epsilon - i(2\tau_1)^{-1}}{\epsilon^2 + (2\tau_1)^{-2}}\Gamma^+(\mathbf{R},t,\epsilon) - \frac{\epsilon + i(2\tau_1)^{-1}}{\epsilon^2 + (2\tau_1)^{-2}}\Gamma^-(\mathbf{R},t,\epsilon)] \ . \tag{2.66}$$

Finally, the parameters α_s and α_d are given by

$$\alpha_s = -[\ln\frac{T_{cs0}}{T} + \psi(\frac{1}{2}) - \psi(\frac{1}{2} + \frac{\rho_s}{2})] \ , \tag{2.67}$$

and

$$\alpha_d = -[\ln\frac{T_{cd0}}{T} + \psi(\frac{1}{2}) - \psi(\frac{1}{2} + \frac{\rho_1}{2})] \ . \tag{2.68}$$

T_{cs0} and T_{cd0} are the critical temperatures of a clean superconductor, which are determined by

$$N(0)V_s\ln(2e^\gamma\omega_D/\pi T_{cs0}) = 1 \ , \tag{2.69}$$

and

$$[N(0)V_d/2]\ln(2e^\gamma\omega_D/\pi T_{cd0}) = 1 \ , \tag{2.70}$$

with γ the Euler constant and ω_D the cut-off frequency. In the presence of impurity scatterings, two transition temperatures are determined by the conditions $\alpha_s(T_{cs}) = 0$ and $\alpha_d(T_{cd}) = 0$. It is very clear that the transition temperature T_{cs} for s-wave order parameter can only be affected by the magnetic impurity scattering while the transition temperature for d-wave order parameter is dominantly affected by the nonmagnetic scattering. The critical temperature of the superconductor is defined by $T_c = \max\{T_{cs}, T_{cd}\}$. We estimate that as long as the d-channel interaction V_d is larger than about three times of the s-channel interaction V_s, the pure d-wave state is stable in the bulk systems without perturbations. The phase diagram of such a system in the absence of external fields and impurities has been previously studied in Ref.[18].

By introducing a formal free energy density

$$\begin{aligned}
f(\mathbf{R},t) = {} & 2\alpha_s|\Delta_s(\mathbf{R},t)|^2 + \alpha_d|\Delta_d(\mathbf{R},t)|^2 + (\frac{v_F}{\pi T})^2\{\frac{1}{2}\chi_{2,1}|\mathbf{\Pi}\Delta_s^*(\mathbf{R},t)|^2 + \frac{1}{4}\chi_{0,3}|\mathbf{\Pi}\Delta_d^*(\mathbf{R},t)|^2 \\
& + \frac{1}{4}\chi_{1,2}[\Pi_x^*\Delta_s(\mathbf{R},t)\Pi_x\Delta_d^*(\mathbf{R},t) - \Pi_y^*\Delta_s(\mathbf{R},t)\Pi_y\Delta_d^*(\mathbf{R},t) + \mathrm{C.C.}]\} \\
& + (\frac{1}{\pi T})^2\{(\chi_{3,0} - \rho_s\chi_{4,0})|\Delta_s(\mathbf{R},t)|^4 + \frac{3}{8}(\chi_{0,3} - \frac{2\rho_1}{3}\chi_{0,4})|\Delta_d(\mathbf{R},t)|^4 \\
& + (2\chi_{2,1} - \rho_1\chi_{2,2})|\Delta_d(\mathbf{R},t)|^2|\Delta_s(\mathbf{R},t)|^2 + \frac{1}{2}\chi_{2,1}[\Delta_d^{*2}(\mathbf{R},t)\Delta_s^2(\mathbf{R},t) \\
& + \Delta_s^{*2}(\mathbf{R},t)\Delta_d^2(\mathbf{R},t)]\}
\end{aligned} \tag{2.71}$$

the TDGL equations (2.61) and (2.62) can be written in a compact way

$$-\frac{1}{\gamma_s}\frac{\partial\Delta_s^*(\mathbf{R},t)}{\partial t}+2\Phi_s(\mathbf{R},t)\Delta_s^*(\mathbf{R},t)=\frac{\delta f(\mathbf{R},t)}{\delta\Delta_s}\ , \tag{2.72a}$$

$$-\frac{1}{\gamma_d}\frac{\partial\Delta_d^*(\mathbf{R},t)}{\partial t}+2\Phi_d(\mathbf{R},t)\Delta_d^*(\mathbf{R},t)=\frac{\delta f(\mathbf{R},t)}{\delta\Delta_d}\ . \tag{2.72b}$$

III. TIME-DEPENDENT CURRENT AND CHARGE DENSITY

A. Current density

The expression for current in "imaginary" frequency space is given as

$$\mathbf{J}(\mathbf{x},\tau)=-\frac{e}{mi}(\nabla_{\mathbf{x}}-\nabla_{\mathbf{x}'})\langle\delta G(\mathbf{x}\tau;\mathbf{x}'\tau^{0+})\rangle|_{\mathbf{x}'\to\mathbf{x}}-\frac{2e^2}{m}\mathbf{A}(\mathbf{x},\tau)\langle\delta G(\mathbf{x}\tau;\mathbf{x}\tau^{0+})\rangle\ , \tag{3.1}$$

where $\langle\delta G_{\alpha\beta}\rangle=\langle G_{\alpha\beta}-G^0_{\alpha\beta}\rangle=\langle\delta G\rangle\delta_{\alpha\beta}$ with $G_{\alpha\beta}$ defined by Eq. (2.10) and the factor 2 arises from the spin sum. Using the similar technique for the gap function, we can divide the current into the normal and anomalous parts, that is,

$$\mathbf{J}_\omega(\mathbf{R})=\mathbf{J}^N_\omega(\mathbf{R})+\mathbf{J}^A_\omega(\mathbf{R},t)\ . \tag{3.2}$$

The normal part is given by

$$\begin{aligned}\mathbf{J}^N_\omega(\mathbf{R})=&(\frac{eT}{mi})\sum_{\epsilon_n\geq0}\int d\mathbf{R}'d\mathbf{r}'d\mathbf{R}''d\mathbf{r}''[e^{-i(\mathbf{R}'-\mathbf{R})\cdot\mathbf{\Pi}^*+(\mathbf{r}'-\mathbf{r})\cdot\nabla_{\mathbf{r}}}\Delta_{\omega_1}(\mathbf{R},\mathbf{r})]\\&\times[e^{i(\mathbf{R}''-\mathbf{R})\cdot\mathbf{\Pi}+(\mathbf{r}''-\mathbf{r})\cdot\nabla_{\mathbf{r}}}\Delta^*_{\omega_2}(\mathbf{R},\mathbf{r})]G^{0(R)}(\mathbf{R}''-\mathbf{r}''/2,\mathbf{R}'-\mathbf{r}'/2;-\epsilon_n)\\&\times\nabla_{\mathbf{r}}\{G^{0(R)}(\mathbf{R}+\mathbf{r}/2,\mathbf{R}'+\mathbf{r}'/2;\epsilon_n)G^{0(R)}(\mathbf{R}''-\mathbf{r}''/2,\mathbf{R}'-\mathbf{r}'/2;\epsilon_n)\}_{\mathbf{r}\to0}\\&-(\frac{eT}{mi})\sum_{\epsilon_n\leq0}\int d\mathbf{R}'d\mathbf{r}'d\mathbf{R}''d\mathbf{r}''[e^{-i(\mathbf{R}'-\mathbf{R})\cdot\mathbf{\Pi}^*+(\mathbf{r}'-\mathbf{r})\cdot\nabla_{\mathbf{r}}}\Delta_{\omega_1}(\mathbf{R},\mathbf{r})]\\&\times[e^{i(\mathbf{R}''-\mathbf{R})\cdot\mathbf{\Pi}+(\mathbf{r}''-\mathbf{r})\cdot\nabla_{\mathbf{r}}}\Delta^*_{\omega_2}(\mathbf{R},\mathbf{r})]G^{0(A)}(\mathbf{R}''-\mathbf{r}''/2,\mathbf{R}'-\mathbf{r}'/2;-\epsilon_n)\\&\times\nabla_{\mathbf{r}}\{G^{0(A)}(\mathbf{R}+\mathbf{r}/2,\mathbf{R}'+\mathbf{r}'/2;\epsilon_n)G^{0(A)}(\mathbf{R}''-\mathbf{r}''/2,\mathbf{R}'-\mathbf{r}'/2;\epsilon_n)\}_{\mathbf{r}\to0}\ .\end{aligned} \tag{3.3}$$

The computation of this part is the same as the static case[15,16] and we give as a result

$$\begin{aligned}\mathbf{J}^N_\omega(\mathbf{R})=&\frac{eE_FN(0)}{m(\pi T)^2}\{\frac{1}{2}\chi_{2,1}\Delta_s^*(\mathbf{R};\omega_2)\mathbf{\Pi}^*\Delta_s(\mathbf{R};\omega_1)+\frac{1}{4}\chi_{0,3}\Delta_d^*(\mathbf{R};\omega_2)\mathbf{\Pi}^*\Delta_d(\mathbf{R};\omega_1)\\&+\frac{1}{4}\chi_{1,2}[\Delta_s^*(\mathbf{R};\omega_2)\Pi_x^*\Delta_d(\mathbf{R};\omega_1)+\Delta_d^*(\mathbf{R};\omega_2)\Pi_x^*\Delta_s(\mathbf{R};\omega_1)]\mathbf{e}_x\\&-\Delta_s^*(\mathbf{R};\omega_2)\Pi_y^*\Delta_d(\mathbf{R};\omega_1)+\Delta_d^*(\mathbf{R};\omega_2)\Pi_y^*\Delta_s(\mathbf{R};\omega_1)]\mathbf{e}_y]\}+\mathrm{C.C.}\ .\end{aligned} \tag{3.4}$$

Here $\mathbf{e}_{x,y}$ is the unit vector along the $x(y)$-direction.

The anomalous part is represented by the diagram shown in Fig. 6. The contribution from the first term is

$$
\begin{aligned}
\mathbf{J}^{A,1}_\omega(\mathbf{R}) &= -(\frac{2e}{mi})\frac{1}{4\pi i}\int d\epsilon(\tanh\frac{\epsilon}{2T}-\tanh\frac{\epsilon-\omega}{2T})\int d\mathbf{x}''[-e\varphi_\omega(\mathbf{x}'')+\frac{e}{m}\mathbf{A}_\omega(\mathbf{x}'')\cdot\mathbf{p}_{\mathbf{x}''}] \\
&\quad\times\nabla_{\mathbf{r}}[G^{0(R)}(\mathbf{R}+\mathbf{r}/2,\mathbf{x}'';\epsilon)G^{0(A)}(\mathbf{x}'',\mathbf{R}-\mathbf{r}/2;\epsilon)]_{\mathbf{r}\to 0} \\
&\approx -(\frac{2e}{mi})\frac{1}{4\pi i}\int d\epsilon(\tanh\frac{\epsilon}{2T}-\tanh\frac{\epsilon-\omega}{2T})\int d\mathbf{x}''\int\frac{d\mathbf{p}_1 d\mathbf{p}_2}{(2\pi)^4}G^{0(R)}(\mathbf{p}_1;\epsilon)G^{0(A)}(\mathbf{p}_2;\epsilon) \\
&\quad\times[-e\varphi_\omega(\mathbf{x}'')+\frac{e}{m}\mathbf{A}_\omega(\mathbf{x}'')\cdot\mathbf{p}_2]\nabla_{\mathbf{r}}[e^{i\mathbf{p}_1\cdot(\mathbf{R}+\mathbf{r}/2-\mathbf{x}'')}e^{i\mathbf{p}_2\cdot(\mathbf{x}''-\mathbf{R}+\mathbf{r}/2)}]_{\mathbf{r}\to 0} \\
&= -(\frac{2e}{mi})\frac{1}{4\pi i}\int d\epsilon(\tanh\frac{\epsilon}{2T}-\tanh\frac{\epsilon-\omega}{2T})\int d\mathbf{x}''\int\frac{d\mathbf{p}}{(2\pi)^2}G^{0(R)}(\mathbf{p};\epsilon)G^{0(A)}(\mathbf{p};\epsilon) \\
&\quad\times\mathbf{p}[-e\varphi_\omega(\mathbf{R})+\frac{e}{m}\mathbf{A}_\omega(\mathbf{R})\cdot\mathbf{p}] \\
&= -\sigma[-i\omega\mathbf{A}_\omega(\mathbf{R})]\ ,
\end{aligned}
\tag{3.5}
$$

where $\sigma = N(0)e^2v_F^2\tau_1 = 2N(0)e^2D$ is the normal-state conductivity. Here we have used the integral

$$
\int d\epsilon\frac{1}{2T}\cosh^{-2}\frac{\epsilon}{2T} = 2\ . \tag{3.6}
$$

Similarly, the contribution of the second term is given by

$$
\begin{aligned}
\mathbf{J}^{A,2}_\omega(\mathbf{R}) &= -(\frac{2e}{mi})\frac{1}{4\pi i}\int d\epsilon\int d\mathbf{x}''(-\Gamma^+_\omega(\mathbf{R},\epsilon)) \\
&\quad\times\nabla_{\mathbf{r}}[G^{0(R)}(\mathbf{R}+\mathbf{r}/2,\mathbf{x}'';\epsilon)G^{0(A)}(\mathbf{x}'',\mathbf{R}-\mathbf{r}/2;\epsilon)]_{\mathbf{r}\to 0} \\
&= -\frac{\sigma\tau_1}{2ie}\nabla\int d\epsilon\Gamma^+_\omega(\mathbf{R},\epsilon) \\
&= -\frac{\sigma\tau_1}{4ie}\nabla\int d\epsilon[\Gamma^+_\omega(\mathbf{R},\epsilon)+\Gamma^-_\omega(\mathbf{R},\epsilon)]\ .
\end{aligned}
\tag{3.7}
$$

By performing an inverse Fourier transform to Eqs. (3.4), (3.5), and (3.7), we obtain the current in real time

$$
\mathbf{J}(\mathbf{R},t) = \mathbf{J}_n(\mathbf{R},t)+\mathbf{J}_s(\mathbf{R},t)\ . \tag{3.8}
$$

Here the normal state current is given by

$$
\mathbf{J}_n(\mathbf{R},t) = -\sigma[\nabla\tilde{\varphi}(\mathbf{R},t)+\frac{\partial\mathbf{A}(\mathbf{R},t)}{\partial t}] \tag{3.9}
$$

where

$$
\tilde{\varphi}(\mathbf{R},t) = \frac{\tau_1}{4ie}\int d\epsilon[\Gamma^+(\mathbf{R},t,\epsilon)+\Gamma^-(\mathbf{R},t,\epsilon)] \tag{3.10}
$$

can be considered as the effective electro-chemical potential for quasi-particles. The supercurrent is given by

$$
\begin{aligned}
\mathbf{J}_s(\mathbf{R},t) &= \frac{eE_FN(0)}{m(\pi T)^2}\{\frac{1}{2}\chi_{2,1}\Delta^*_s(\mathbf{R},t)\mathbf{\Pi}^*\Delta_s(\mathbf{R},t)+\frac{1}{4}\chi_{0,3}\Delta^*_d(\mathbf{R},t)\mathbf{\Pi}^*\Delta_d(\mathbf{R},t) \\
&\quad+\frac{1}{4}\chi_{1,2}[\Delta^*_s(\mathbf{R},t)\Pi^*_x\Delta_d(\mathbf{R},t)+\Delta^*_d(\mathbf{R},t)\Pi^*_x\Delta_s(\mathbf{R},t)]\mathbf{e}_x \\
&\quad-\Delta^*_s(\mathbf{R},t)\Pi^*_y\Delta_d(\mathbf{R},t)+\Delta^*_d(\mathbf{R},t)\Pi^*_y\Delta_s(\mathbf{R},t)]\mathbf{e}_y]\}+\mathrm{C.C.} \\
&= -\frac{N(0)}{4}\frac{\delta f(\mathbf{R},t)}{\delta\mathbf{A}}\ .
\end{aligned}
\tag{3.11}
$$

B. Charge density

The charge density in the "imaginary" time space is defined by

$$\rho(\mathbf{x},\tau) = -2e\langle G(\mathbf{x}\tau,\mathbf{x}\tau^{+0})\rangle \ . \tag{3.12}$$

After the analytical continuation, we have

$$\begin{aligned}\rho_\omega(\mathbf{x}) &= -2eT\sum_\epsilon G_{\epsilon,\epsilon-\omega}(\mathbf{x},\mathbf{x})\\ &= \rho^N_\omega(\mathbf{R}) + \rho^A_\omega(\mathbf{R}) \ ,\end{aligned} \tag{3.13}$$

with

$$\begin{aligned}\rho^N_\omega(\mathbf{R}) &= (-2e)\frac{1}{4\pi i}\int d\epsilon \tanh\frac{\epsilon}{2T}\int d\mathbf{x}_1(-e\varphi_\omega(\mathbf{x}_1)\\ &\quad\times[G^{0(R)}(\mathbf{x},\mathbf{x}_1;\epsilon+\omega)G^{0(R)}(\mathbf{x}_1,\mathbf{x};\epsilon) - G^{0(A)}(\mathbf{x},\mathbf{x}_1;\epsilon)G^{0(A)}(\mathbf{x}_1,\mathbf{x};\epsilon-\omega)\\ &= (-2e)T\{\sum_{\epsilon_n\geq 0}\int\frac{d\mathbf{p}}{(2\pi)^2}[G^{0(R)}(\mathbf{p},\epsilon_n)]^2 - \sum_{\epsilon_n\leq 0}\int\frac{d\mathbf{p}}{(2\pi)^2}[G^{0(A)}(\mathbf{p},\epsilon_n)]^2\\ &= -e^2N(0)\varphi_\omega(\mathbf{R})\int d\xi\{[\mathcal{P}\frac{1}{i\pi\xi}+\delta(\xi)] + [-\mathcal{P}\frac{1}{i\pi\xi}+\delta(\xi)]\}\\ &= -2N(0)e^2\varphi_\omega(\mathbf{R}) \ ,\end{aligned} \tag{3.14}$$

and

$$\begin{aligned}\rho^A_\omega(\mathbf{R}) &= -(\frac{e}{mi})\frac{1}{4\pi i}\int d\epsilon(\tanh\frac{\epsilon}{2T} - \tanh\frac{\epsilon-\omega}{2T})\int d\mathbf{x}''(-e\varphi_\omega(\mathbf{x}'') + e\mathbf{A}_\omega(\mathbf{x}'')\cdot\mathbf{p}_{\mathbf{x}''})\\ &\quad\times[G^{0(R)}(\mathbf{R}+\mathbf{r}/2,\mathbf{x}'';\epsilon)G^{0(A)}(\mathbf{x}'',\mathbf{R}-\mathbf{r}/2;\epsilon)]_{\mathbf{r}\to 0}\\ &= (-2e)\frac{1}{4\pi i}\int d\epsilon(-\Gamma^+_\omega(\mathbf{R},\epsilon))\int\frac{d\mathbf{p}}{(2\pi)^2}G^{0(R)}(\epsilon,\mathbf{p})G^{0(A)}(\epsilon,\mathbf{p})\\ &= -\frac{iN(0)e\tau_1}{2}\int d\epsilon[\Gamma^+_\omega(\mathbf{R},\epsilon) + \Gamma^-_\omega(\mathbf{R},\epsilon)]\\ &= 2e^2N(0)\tilde{\varphi}_\omega(\mathbf{R}) \ .\end{aligned} \tag{3.15}$$

After an inverse Fourier transform, we have the charge density in real time space

$$\rho(\mathbf{R},t) = 2e^2N(0)[\tilde{\varphi}(\mathbf{R},t) - \varphi(\mathbf{R},t)] \ . \tag{3.16}$$

From Eqs. (3.8), (3.16), and (2.59) follows the continuity equation $\nabla\cdot\mathbf{J} + \partial\rho/\partial t = 0$.

IV. DISCUSSIONS AND SUMMARY

Combined with the Maxwell equations, which couple $\mathbf{A}$ and φ with $\mathbf{J}$ and ρ, Eqs. (2.72), (3.8), (3.16), together with Eqs. (2.59) constitute a complete set of coupled time-dependent Ginzburg-Landau equations, which are our main results. Several features of the above results deserve special attention: It is well known that depairing of s-wave superconductors are due only to magnetic impurities. However, nonmagnetic impurities can have direct depairing

effects on unconventional d-wave pairing state. Similarly, the relaxation of the s-wave order parameter is influenced only by magnetic impurities. Therefore, the magnetic impurities as pair-breakers are essential in the derivation of the corresponding TDGL equations for conventional s-wave superconductors.[7] However, nonmagnetic impurities acting as depairing centers can directly affect the relaxation of the d-wave order parameter. Interestingly, for a mixed d- and s-wave symmetry superconductor with a high concentration of magnetic and nonmagnetic impurities such that $\tau_1 T_c \ll 1$ and $\tau_s T_c \ll 1$, we have $\gamma_d^{-1} \approx \tau_1$, $\gamma_s^{-1} \approx \tau_s$. In this limit, the TDGL equations for the order parameters, Eqs. (2.72a) and (2.72b), become

$$-\tau_s[\frac{\partial}{\partial t} + 2ie\tilde{\varphi}(\mathbf{R},t)]\Delta_s^*(\mathbf{R},t) = \frac{\delta f(\mathbf{R},t)}{\delta \Delta_s} \; , \tag{4.1}$$

$$-\tau_1[\frac{\partial}{\partial t} + 2ie\tilde{\varphi}(\mathbf{R},t)]\Delta_d^*(\mathbf{R},t) = \frac{\delta f(\mathbf{R},t)}{\delta \Delta_d} \; , \tag{4.2}$$

where the coefficients α_s,α_d, and $\chi_{m,n}$ can also be simplified, but are not explicitly given here.

These set of TDGL equations valid under the strong gaplessness conditions are similar in form to that postulated phenomenologically[5] except that the relaxation parameters obtained here are γ_s $(= \tau_s^{-1})$ and γ_d $(= \tau_1^{-1})$ and the usual scalar potential φ is replaced by the electrochemical potential $\tilde{\varphi}$. Therefore, the phenomenological TDGL equations are at most valid when the superconductor is very dirty with also a high concentration of magnetic impurities. If the superconductor is doped only with high density of nonmagnetic impurities ($\tau_s \gg \tau_1$), the TDGL equation (2.72b) for d-wave component is reduced to Eq. (4.2) while the relaxation parameter involved in the equation for s-wave component becomes $\gamma_s^{-1} \approx \pi/4T_c$. In this case, $\gamma_s \ll \gamma_d$ and the TDGL equations for both components are quite asymmetric. Of particular interest, if $T_{cs} < T < T_{cd}$, due to a mixed gradient coupling of the s- and d-wave components, the s-wave order parameter with four-lobe structure is induced near the d-wave vortex core, and the overall structure of an individual vortex is fourfold symmetric. Numerical simulation,[5] where the same relaxation rate ($\gamma_s = \gamma_d = \gamma$) was assumed for two components, showed an intrinsic contribution to the Hall angle caused by the lack of complete rotational symmetry in d-wave superconductivity. In the case $\tau_s \gg \tau_1$, we could have $\gamma_s \ll \gamma_d$ and the d-wave order parameter relaxes much faster than the s-wave component. Under this condition we expect that the s-wave component will not be able to follow the motion of the d-wave vortex and novel phenomenon may appear in the flux dynamics. Even when $\tau_1 T_c$ and $\tau_s T_c$ are both small, the condition $\gamma_d = \gamma_s$ used in Ref.[5] would require the assumption that the non-spin-flip interaction $U_1 = 0$, which as judged from the studies on conventional s-wave superconductors,[7] may well be not justifiable.

In summary, we have derived the TDGL equations for superconductors with mixed d-wave and s-wave symmetry assuming a weak gapless condition for both types of order parameters. From this derivation, the unknown coefficients for the TDGL equations postulated phenomenologically have been ascertained. This set of TDGL equations can be used as the starting point for the study of the vortex dynamics in superconductors with the mixed d- and s-wave symmetry, or even extended to study other transport coefficients. In particular, the issue of how the dynamic properties of vortices are influenced by the admixture of an induced s-wave component with the dominant d-wave component of the order parameter as

well as their different responses to the impurity scatterings can be studied systematically. The TDGL equations for d-wave superconductors with on-site s-wave repulsive interaction can be similarly obtained by using the Padé approximation,[4] and we find that the main conclusion of the present paper still remains unchanged. This result together with a detailed derivation will be presented elsewhere. Finally, we remark that the present derivation has not included the effects of electron-electron (actually hole-hole), electron-phonon, and electron-"any magnetic excitation" scatterings, which might be more important in high-T_c superconductors than in conventional low-T_c superconductors. Whereas such inelastic scatterings are far from being easy to incorporate within the present framework, we think that their dominant qualitative and perhaps semi-quantitative effects can be taken into account phenomenologically by adding a term $1/\tau_E$ to the diffusion operator $\partial/\partial t - D\nabla^2$, where τ_E stands for an inelastic relaxation time (assuming that the weak gaplessness conditions are still satisfied). Consistent with such an approach one should regard τ_1 and τ_s as effective, including also some effects of the inelastic scatterings.

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FIGURES

FIG. 1. Ladder-type diagram leading to $I(\omega, \mathbf{k})$. The momenta and frequencies for the solid (electron) lines in the upper part are $\mathbf{p}$ and ϵ, and those for the solid lines in the lower part are $\mathbf{p} - \mathbf{k}$ and $\epsilon - \omega$.

FIG. 2. Impurity-averaged diagrams leading to the diffusion equation for Γ^{+}. The thick wavy lines correspond to $I(\omega, \mathbf{k})$ shown in Fig. 1. The thin wavy line corresponds to the vertex interaction with the electromagnetic field. The triangle represents the order parameter.

FIG. 3. Impurity-averaged diagrams for kernel Q_1. $\tilde{\Delta}$ and $\tilde{\Delta}^*$ are both the vertex-renormalized order parameters in the upper part.

FIG. 4. Impurity-averaged diagrams for kernel Q_3. $\tilde{\Delta}$ is the vertex-renormalized order parameter in the upper part and $\tilde{\Delta}^*$ the vertex-renormalized order parameter in the lower part.

FIG. 5. Impurity-averaged diagrams leading to the anomalous part in the TDGL equation for order parameter. $\Gamma^{\pm}$ are given by the type of diagrams shown in Fig. 2.

FIG. 6. Impurity-averaged diagrams leading to the anomalous current density.